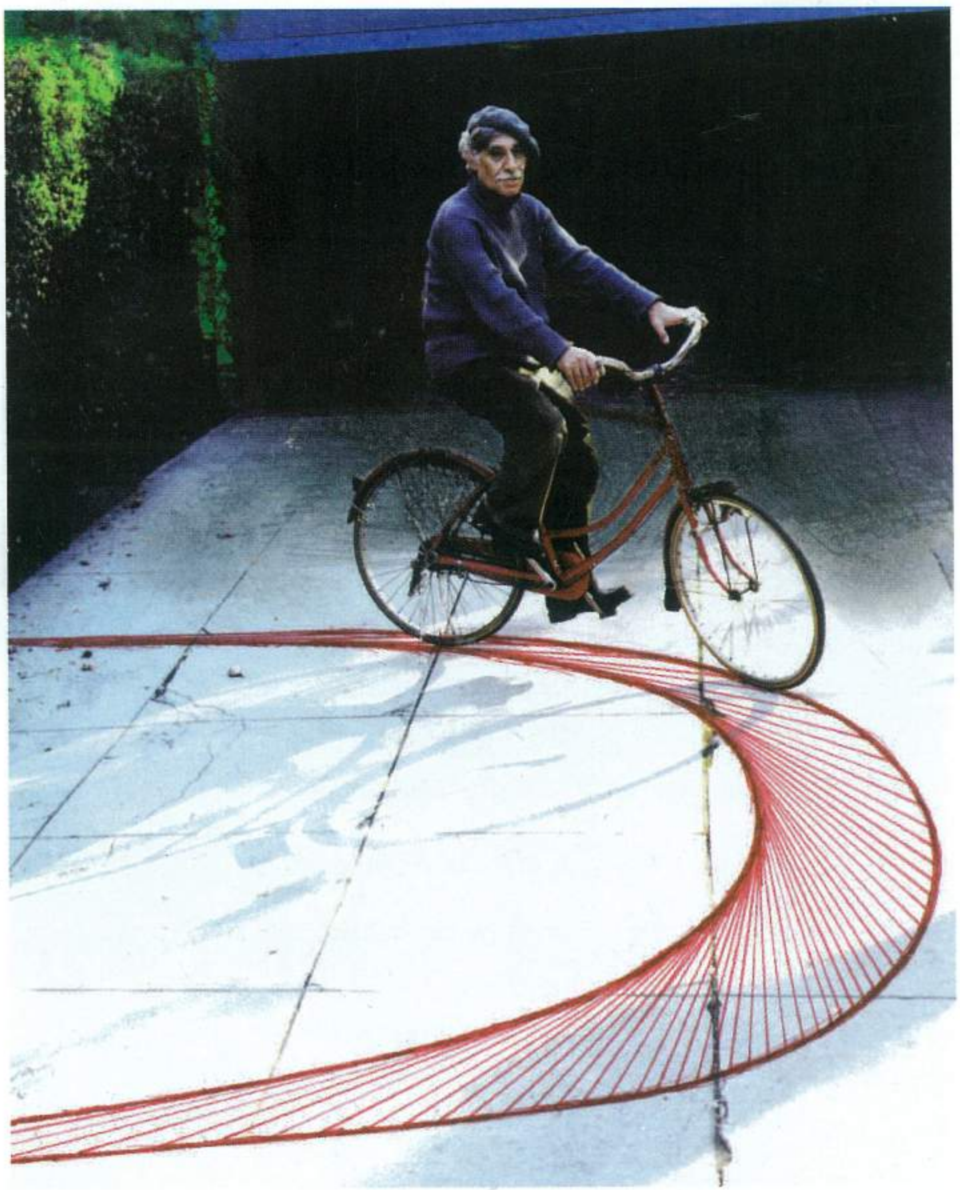
Visual Calculus



What is the area traced out between the front and back wheels?

Mamikon Mnatsakanian

Calculating Areas

Retro- or steam-punk computing :-

How would we calculate areas or volumes if integral calculus had never been invented?

Mostly special cases – but includes many of the interesting curves people have named.

A little history first.

Archimedes c. 287– 212 BC

Rigorous value of π with a lower and upper bound.

Volume and area of sphere, and the volume and centre of gravity of the parabaloid and hemisphere.

Various ingenious geometric theorems

Plus more practical inventing and constructing as well!

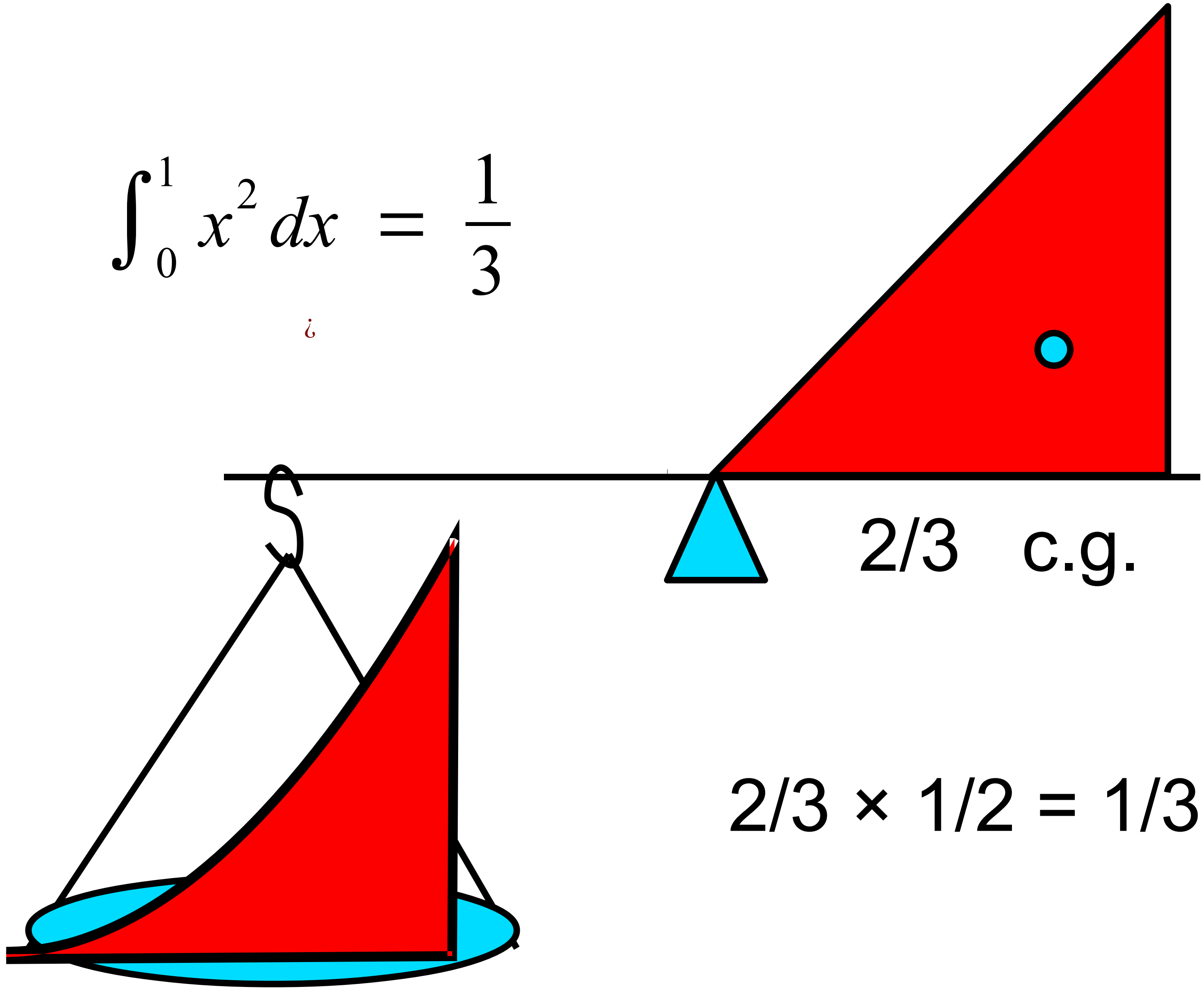
The Method

Devised 'The Method', lost till recently
Found in the Archimedes Palimpest

- Equilibrium of Planes
- Spiral Lines
- Measurement of a Circle
- On the Sphere and Cylinder
- On Floating Bodies
- The Method of Mechanical Theorems
- Stomachion

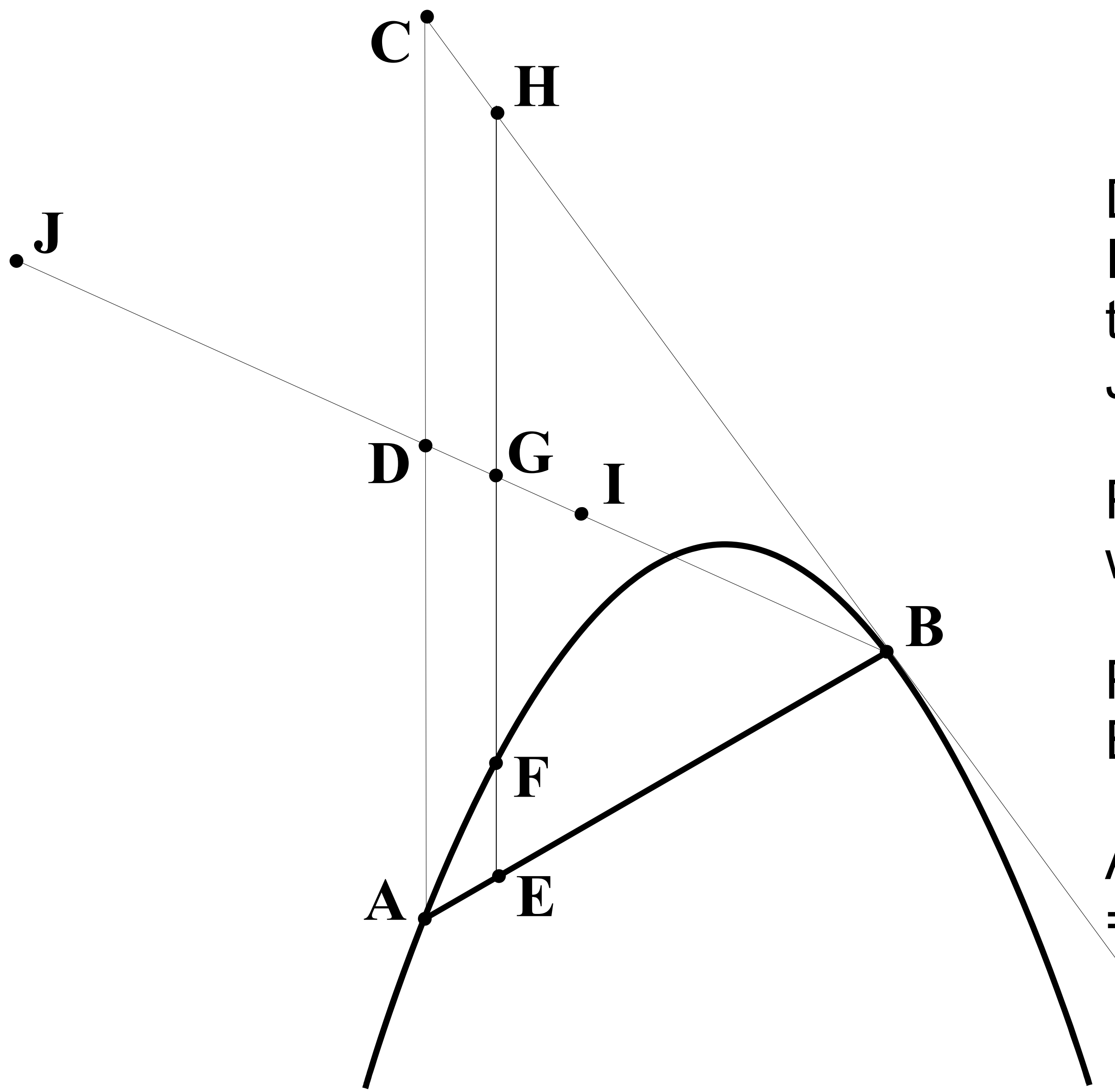
Area of a Parabola

$$\int_0^1 x^2 \, dx = \frac{1}{3}$$



2/3 × 1/2 = 1/3

Area of a Parabola



D - Fulcrum
I – centre of gravity of triangle ABC
JD = DB

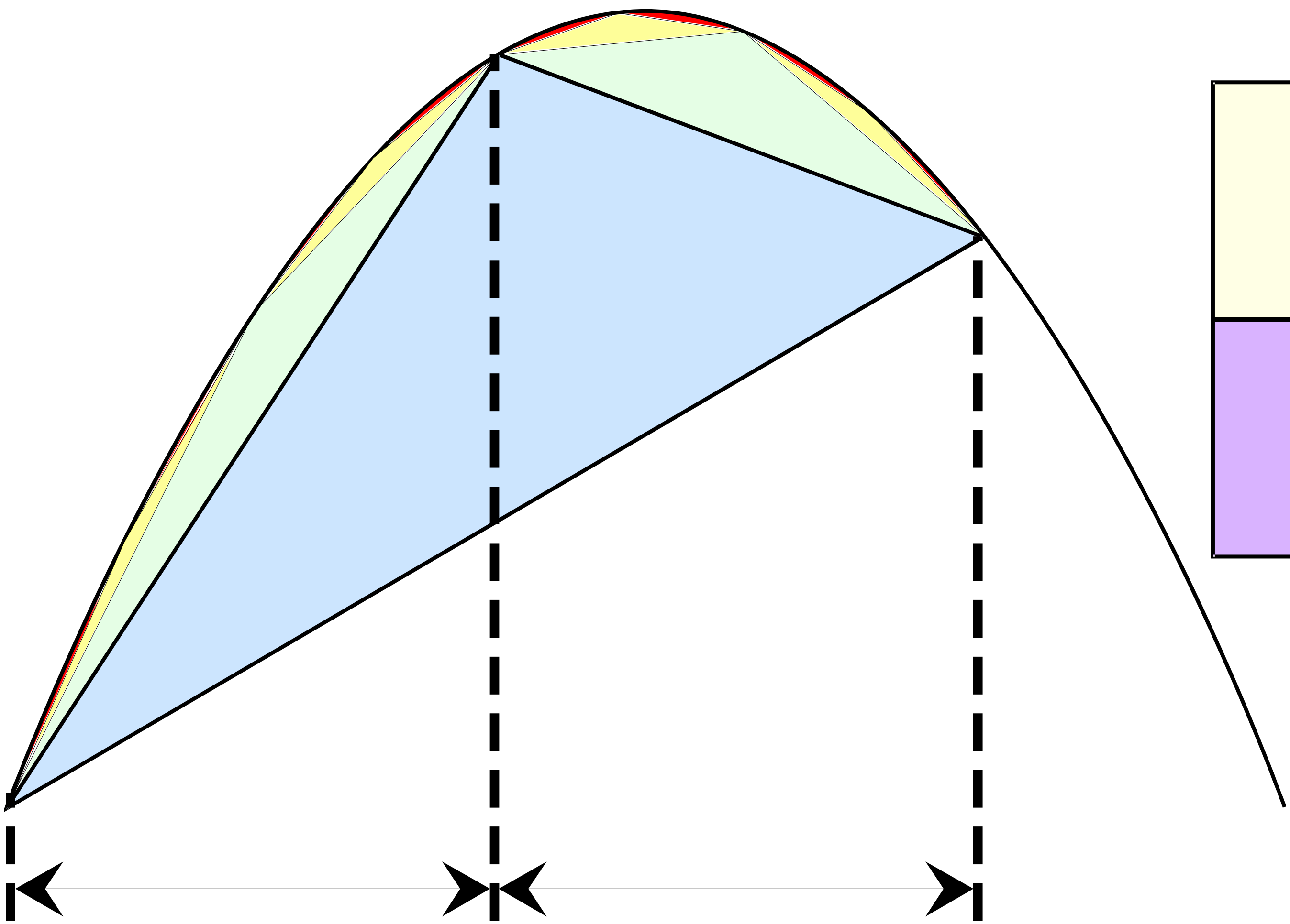
Parabola AFB at J would balance ABC

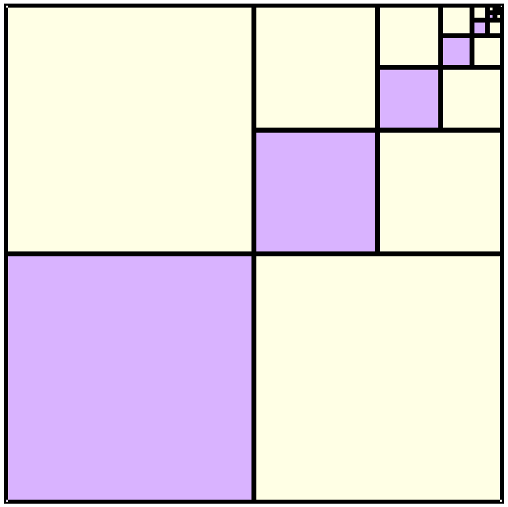
Proved that
EF:GD = EH:JD

Area of parabola AFB = 1/3 area of ABC

Area of a Parabola

Official solution in 'The Quadrature of the Parabola'





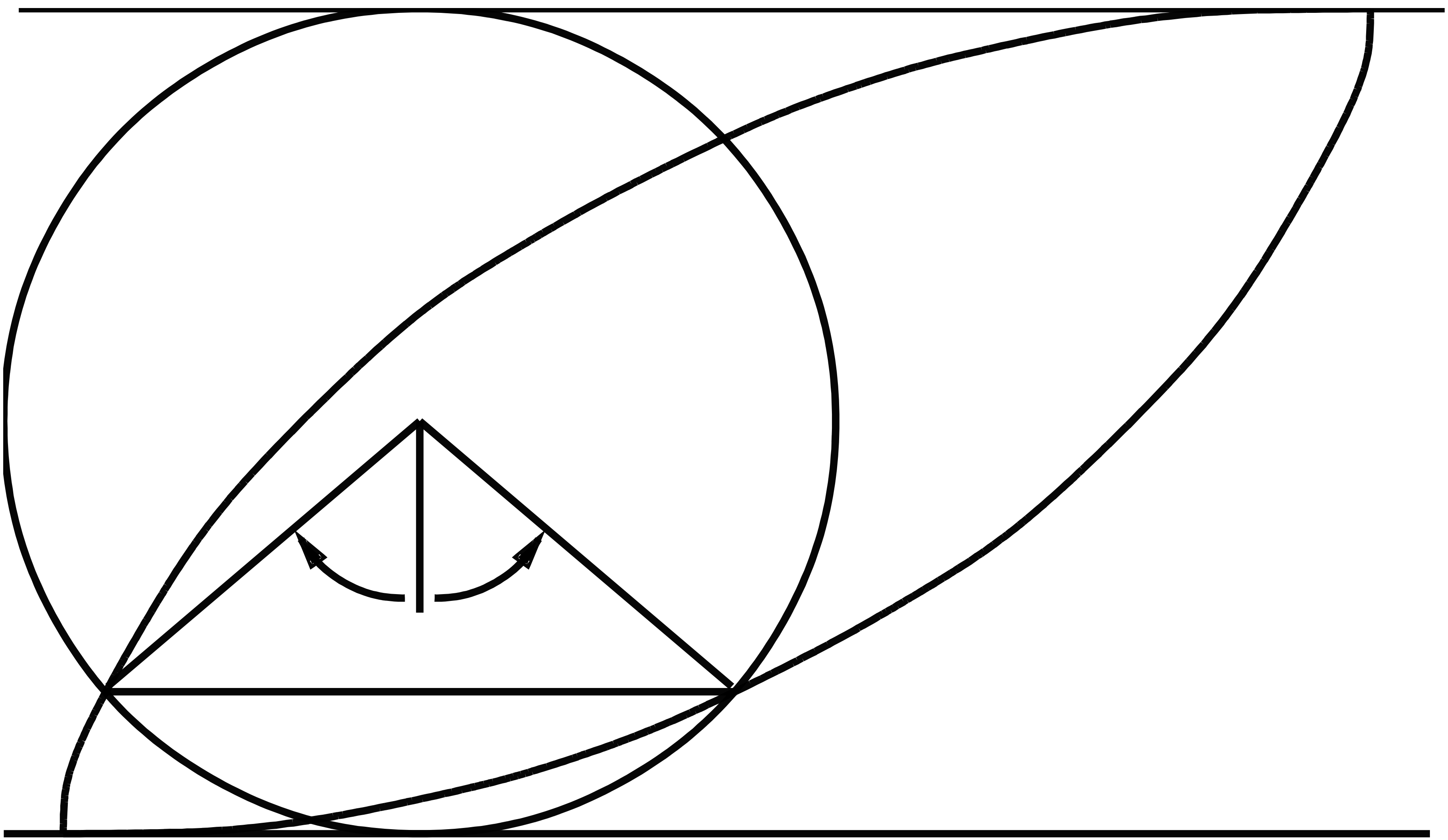
Cavalieri 1598 –1647

“Geometry, developed by a new method through the indivisibles of the continua”.

Cavalieri's Principle

The area/volumes of two objects are equal if the lengths/areas of their cross-sections are equal when they are both cut by a set of parallel lines/planes.

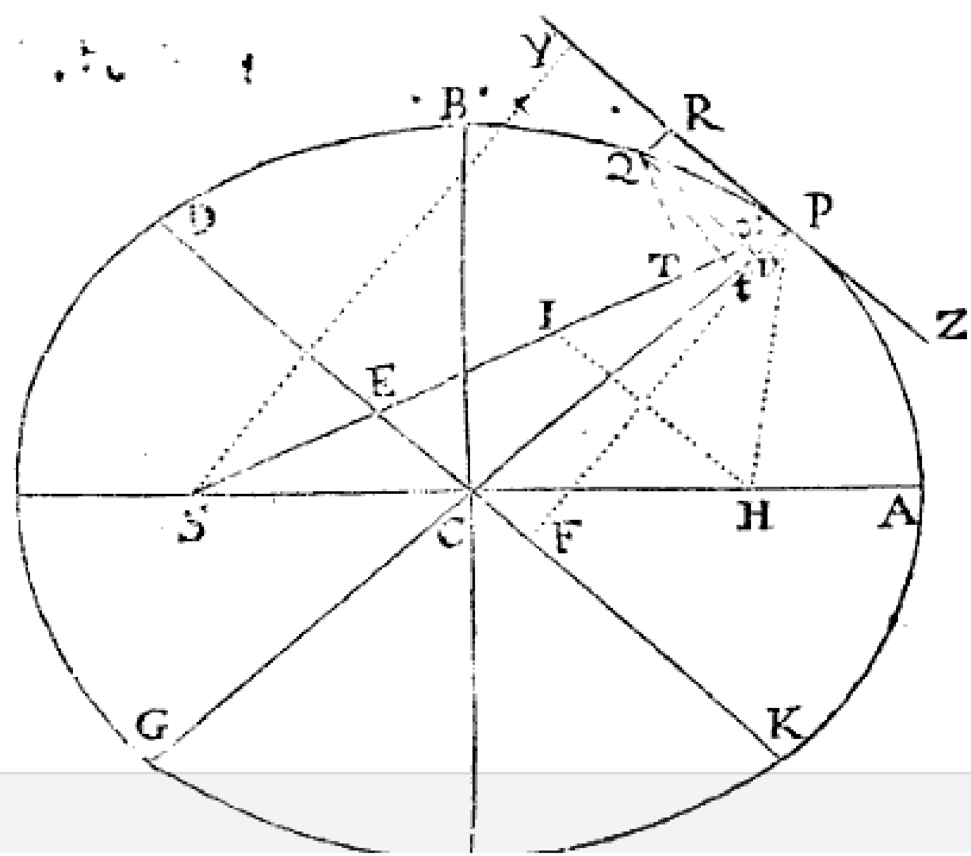
Area of a Cycloid



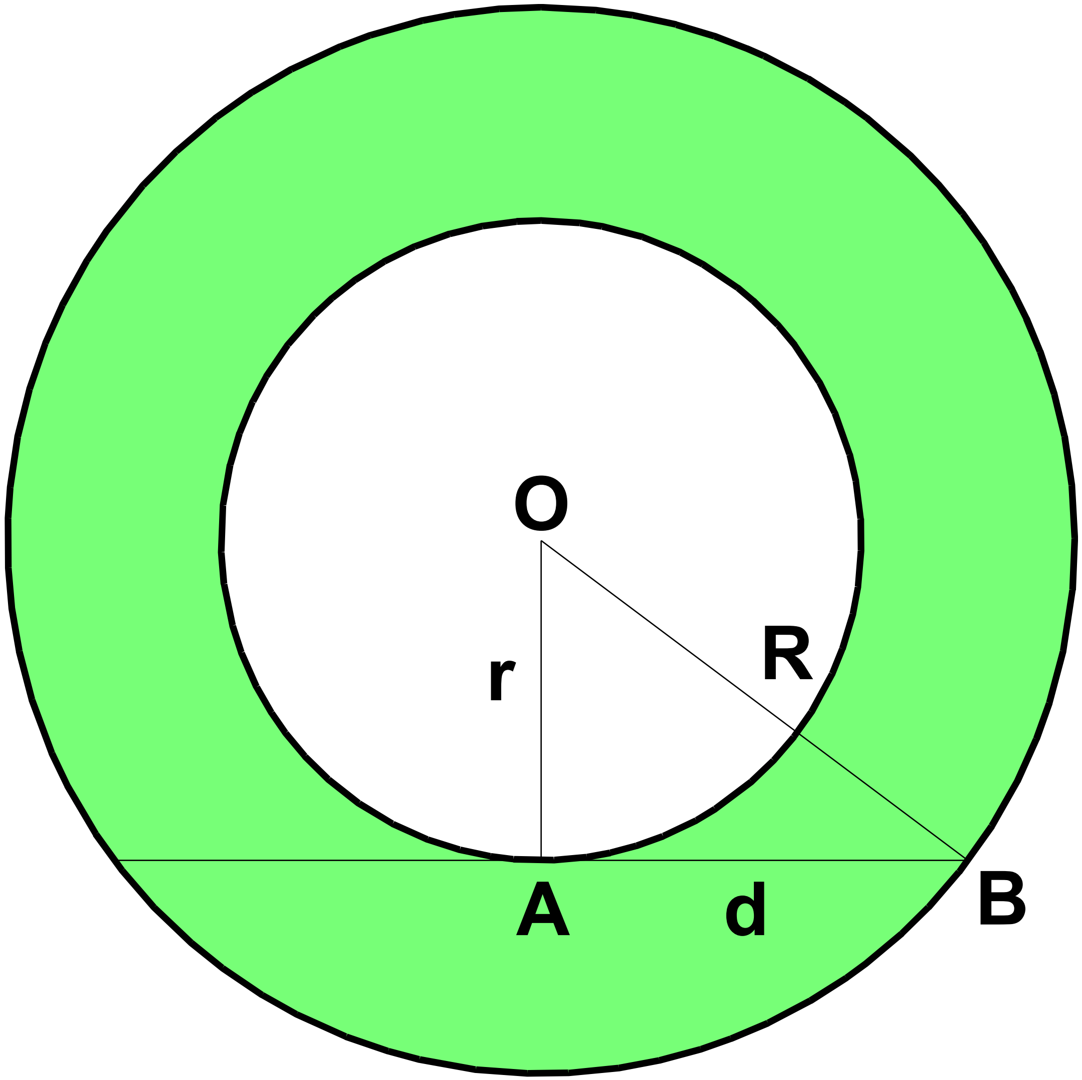
Newton

Revolvatur corpus in Ellipſi: Requiritur lex vis centripetæ tendentis ad umbilicum Ellipſeos.

Eſto Ellipſeos ſuperioris umbilicus S. Agatur *SP* ſecans Ellipſeos tum diametrum *DK* in *E*, tum ordinatim applicatam *Qv* in *x*, & compleatur parallelogrammum *Qx* P *R*. Patet *E* P æqualem eſſe ſemiaxi majori *AC*, eo quod acta ab altero Ellipſeos umbilico H linea *HI* ipſi *EC* parallela, (ob æquales *CS*, *CH*) æquentur ES, *EI*, adeo ut EP ſemiſumma ſit ipſarum P S, P *I*, id eſt (ob parallelas *HI*, P *R* & angulos æquales *I* P *R*, *H* P *Z*) ipſo-



Area of an Annulus



$Area = \pi R^2 - \pi r^2$

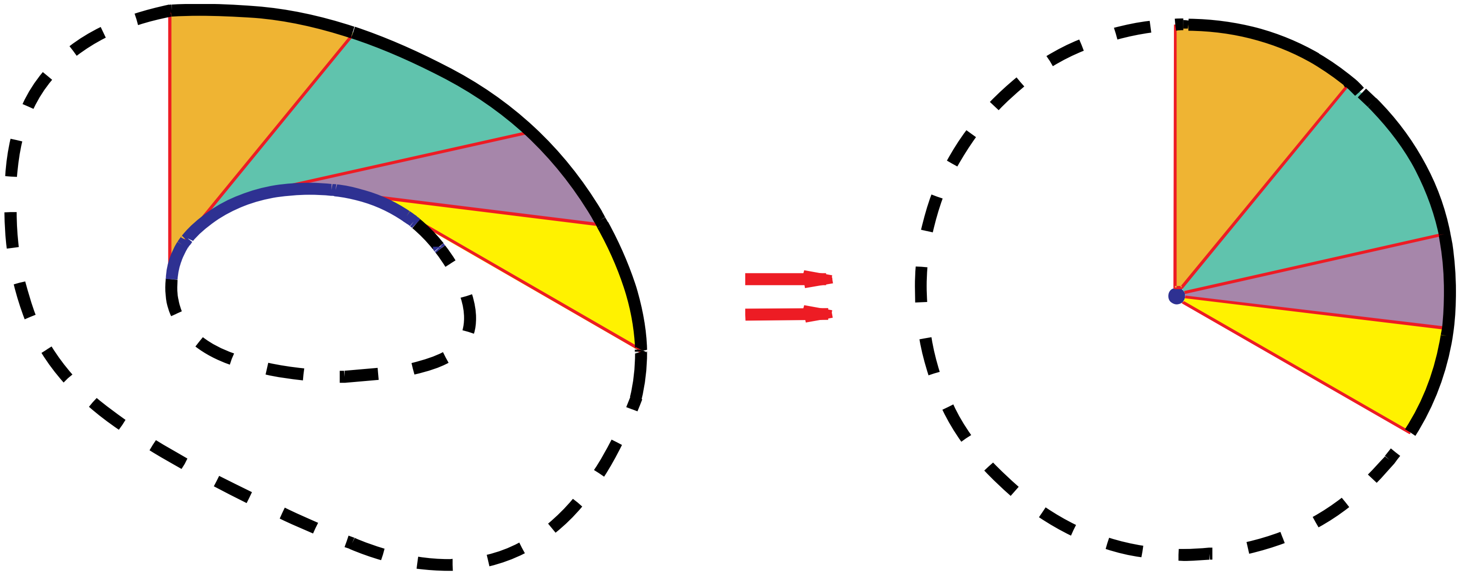
$d^2 = R^2 - r^2$

$\Rightarrow$

$Area = \pi d^2$

Mamikon's theorem

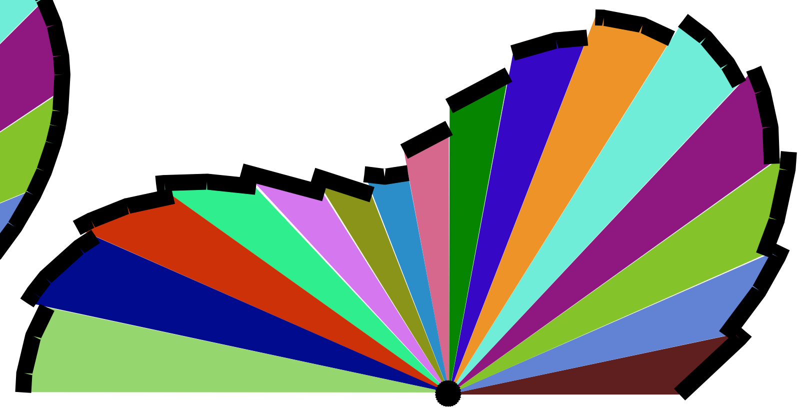
The area of a tangent sweep is equal to the area of its tangent cluster.



A few terms

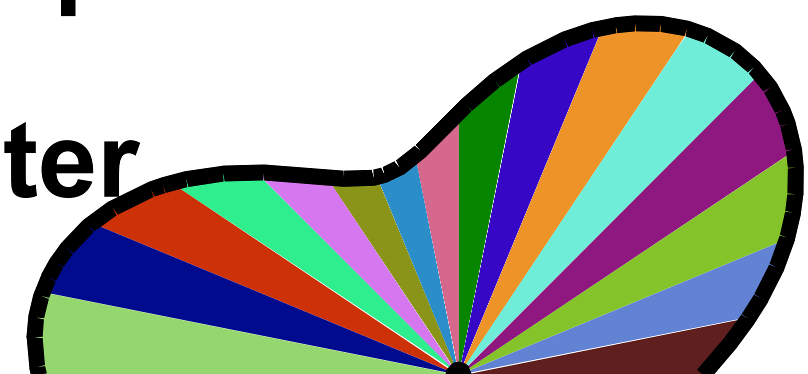
free end curve

tangency sweep



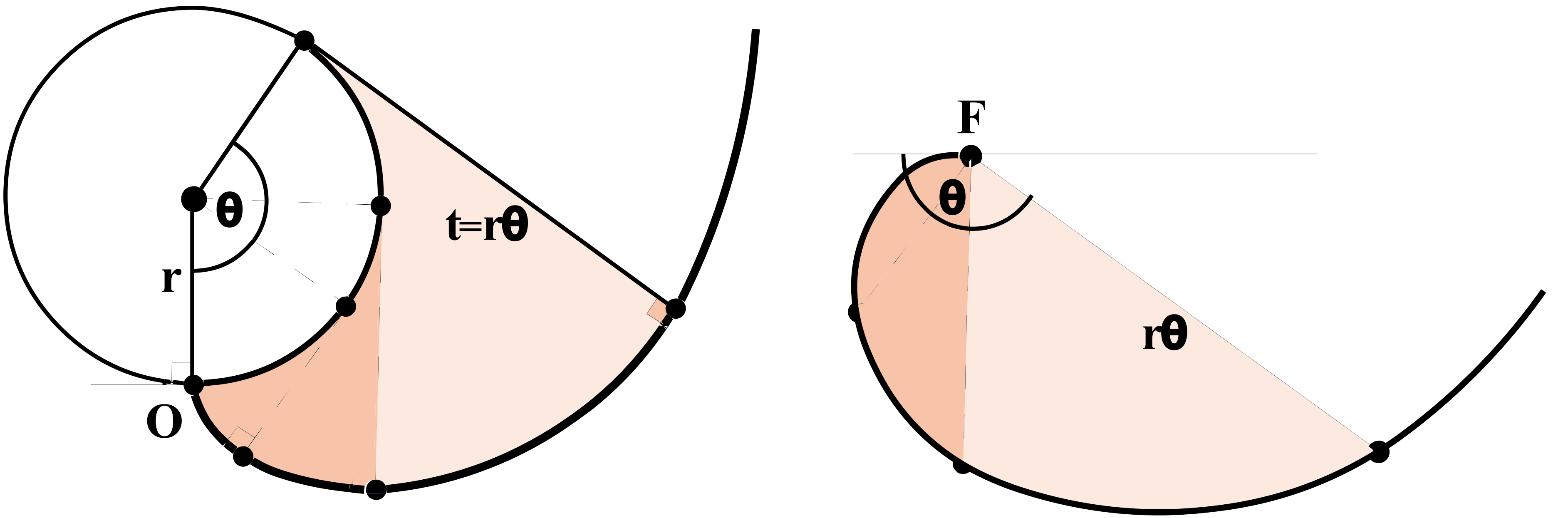
Tangent sweep

Tangent cluster



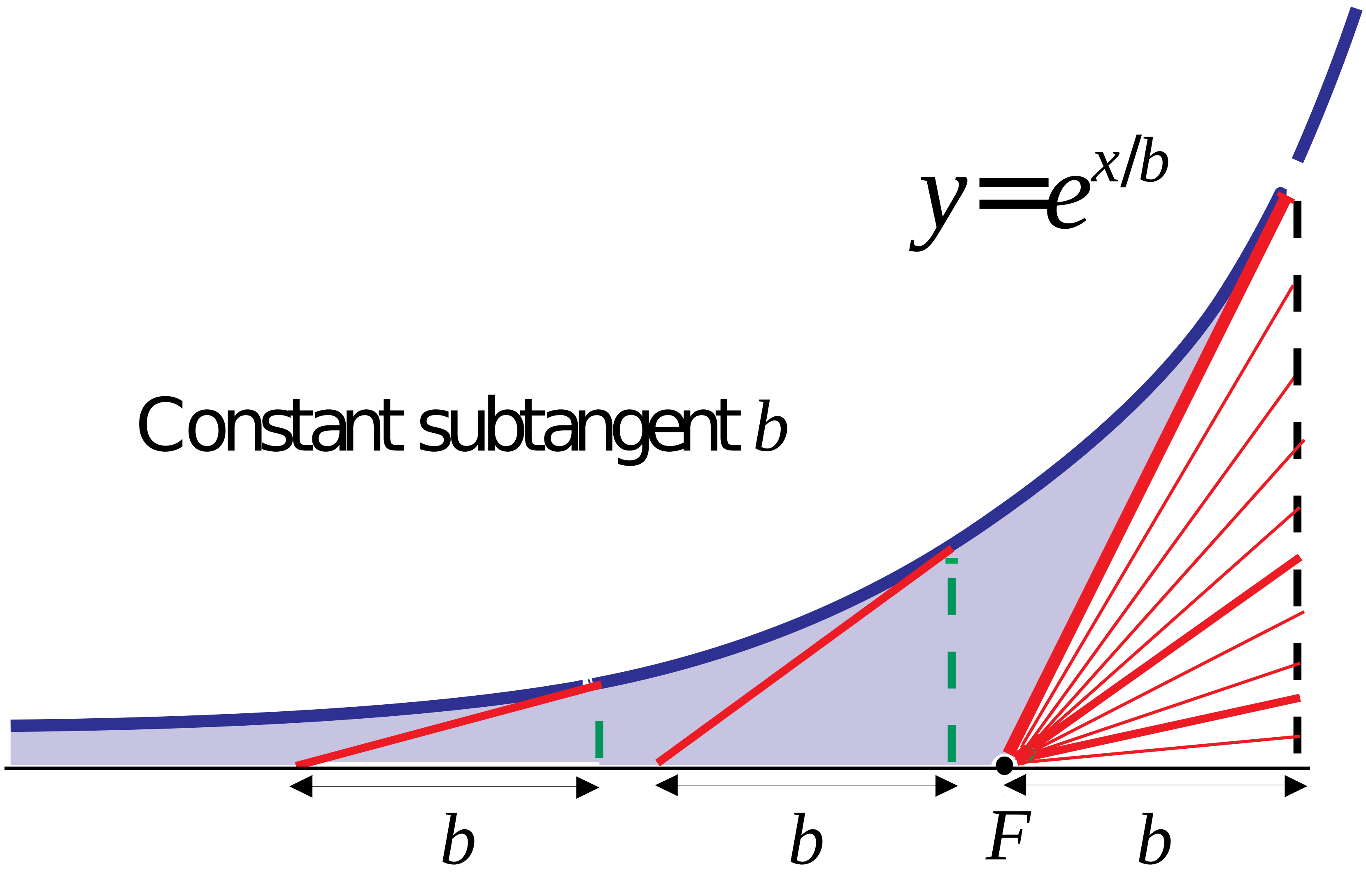
Archimedean Spiral

Same area as the involute of the circle

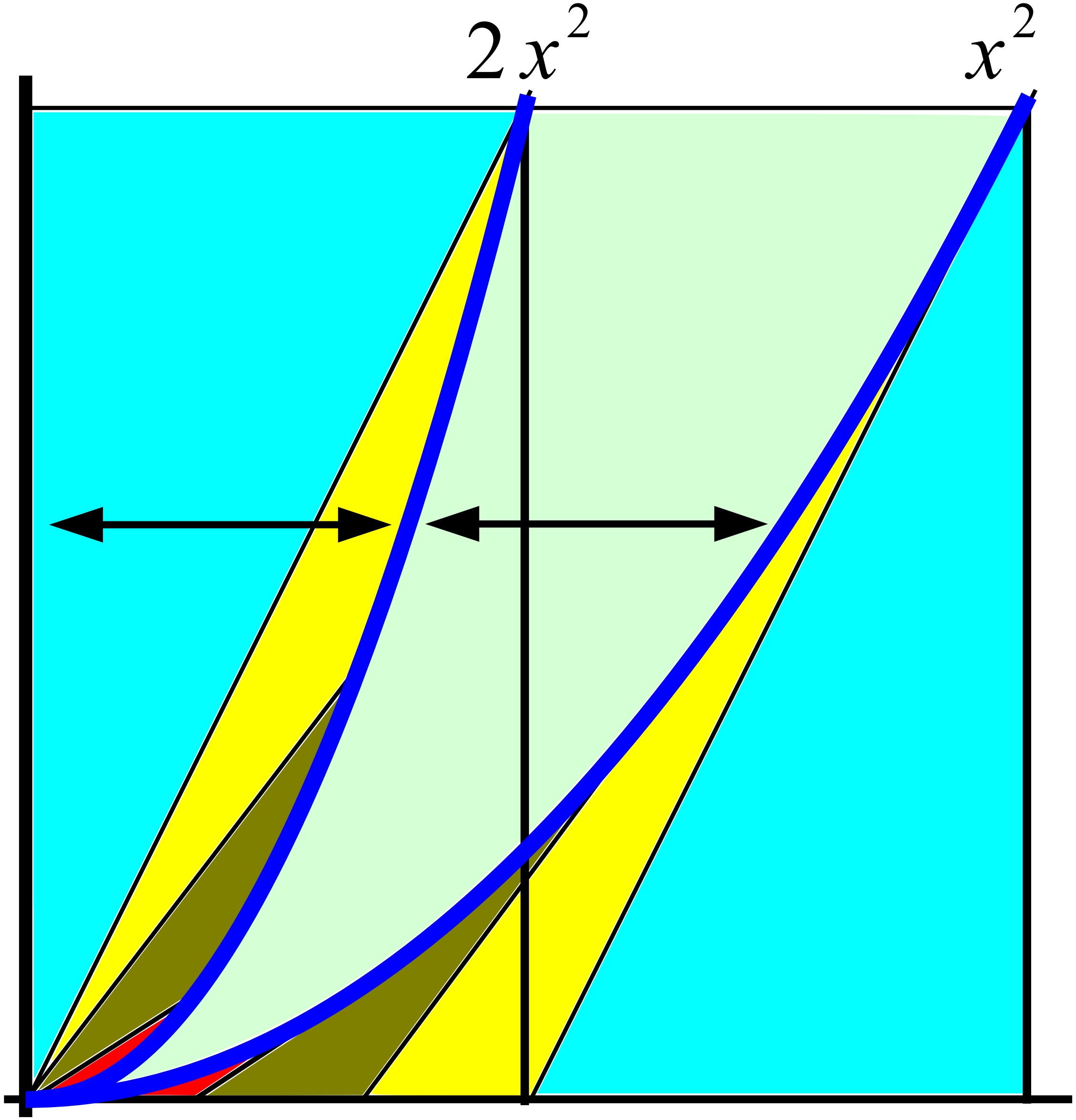


$$Area = \frac{\theta^2}{2}$$

Exponential curve



Parabola



Subtangent at x is x/2

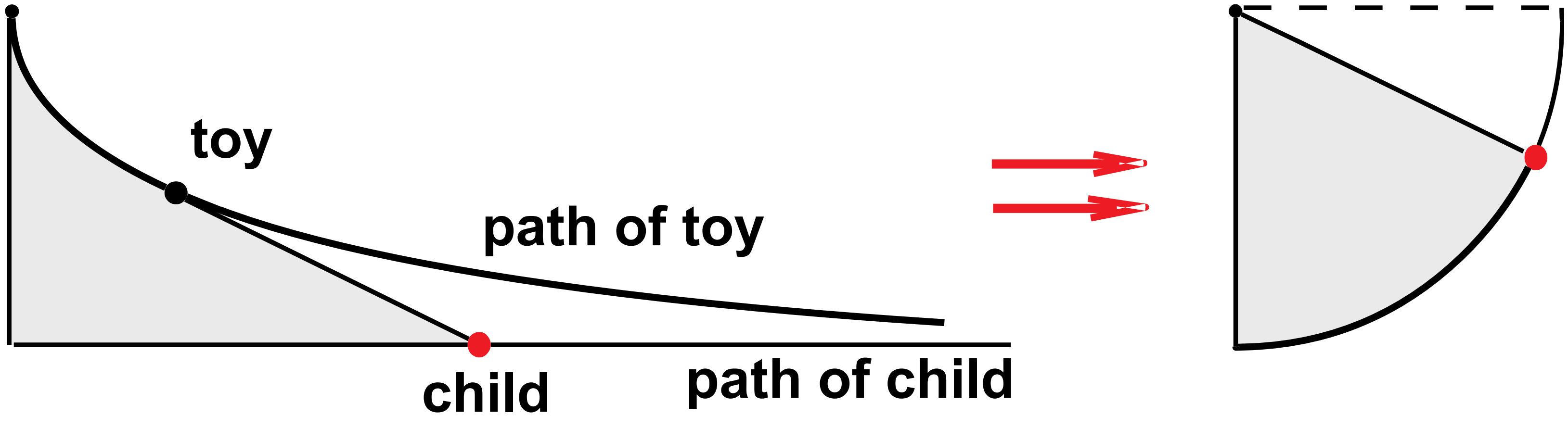
Three equal areas

Therefore area under parabola is:-

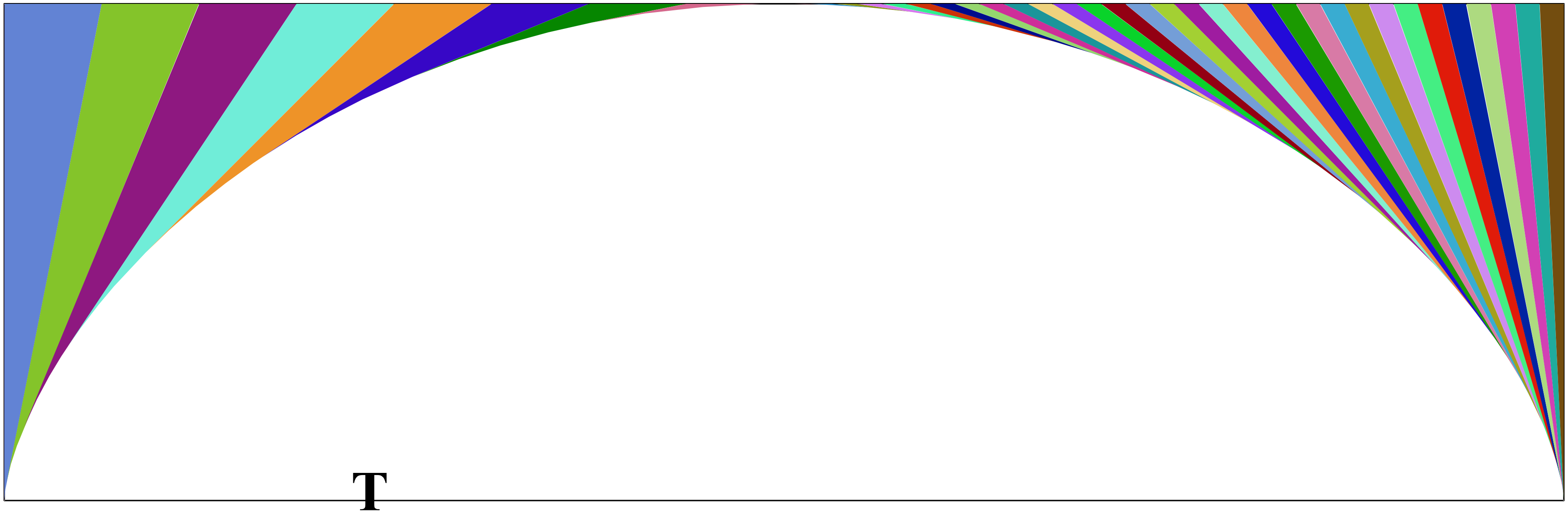
one third the area of the enclosing rectangle

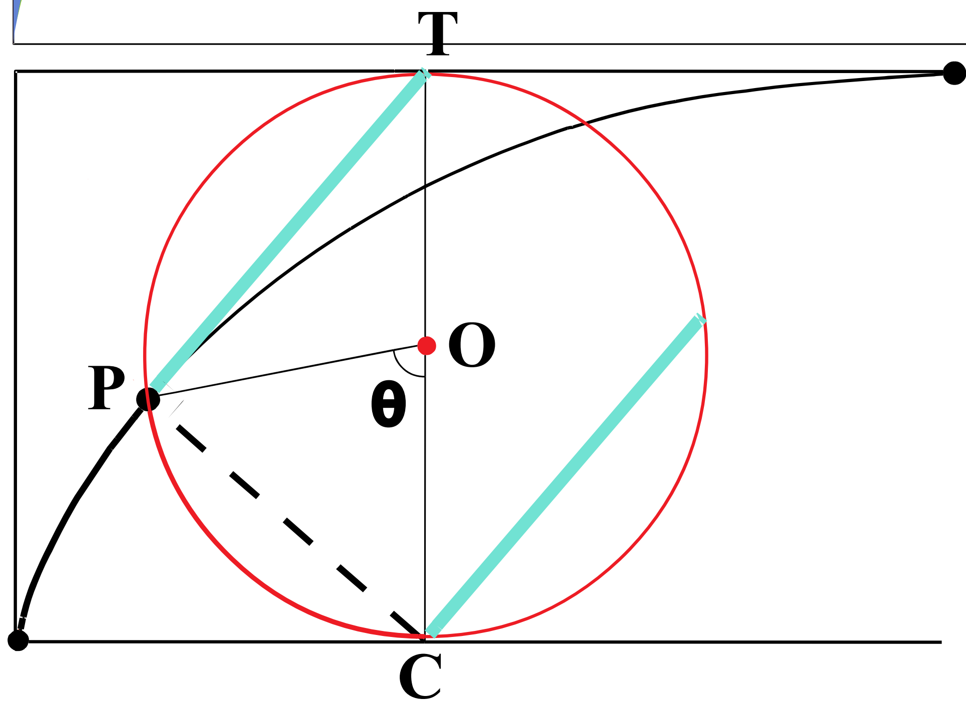
The Tractrix

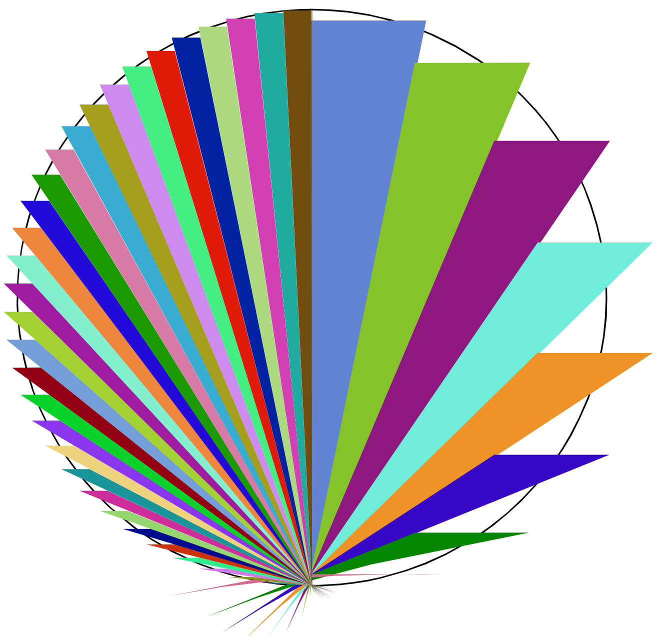
Boy pulling a cart.
String is constant length



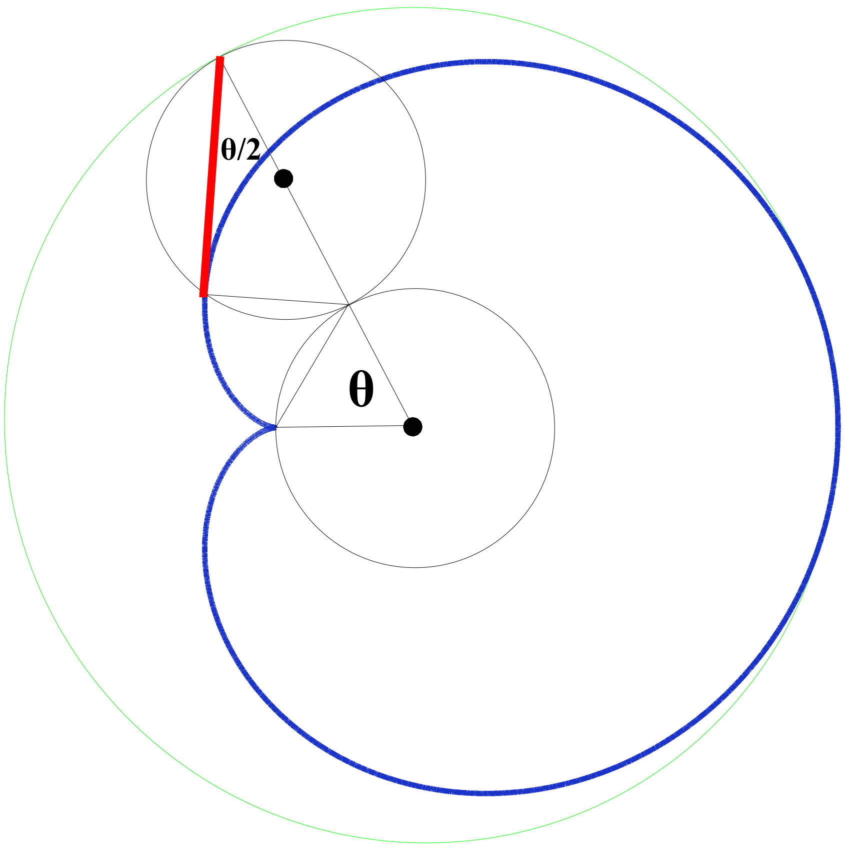
The area of the Cycloid again







Cardioid



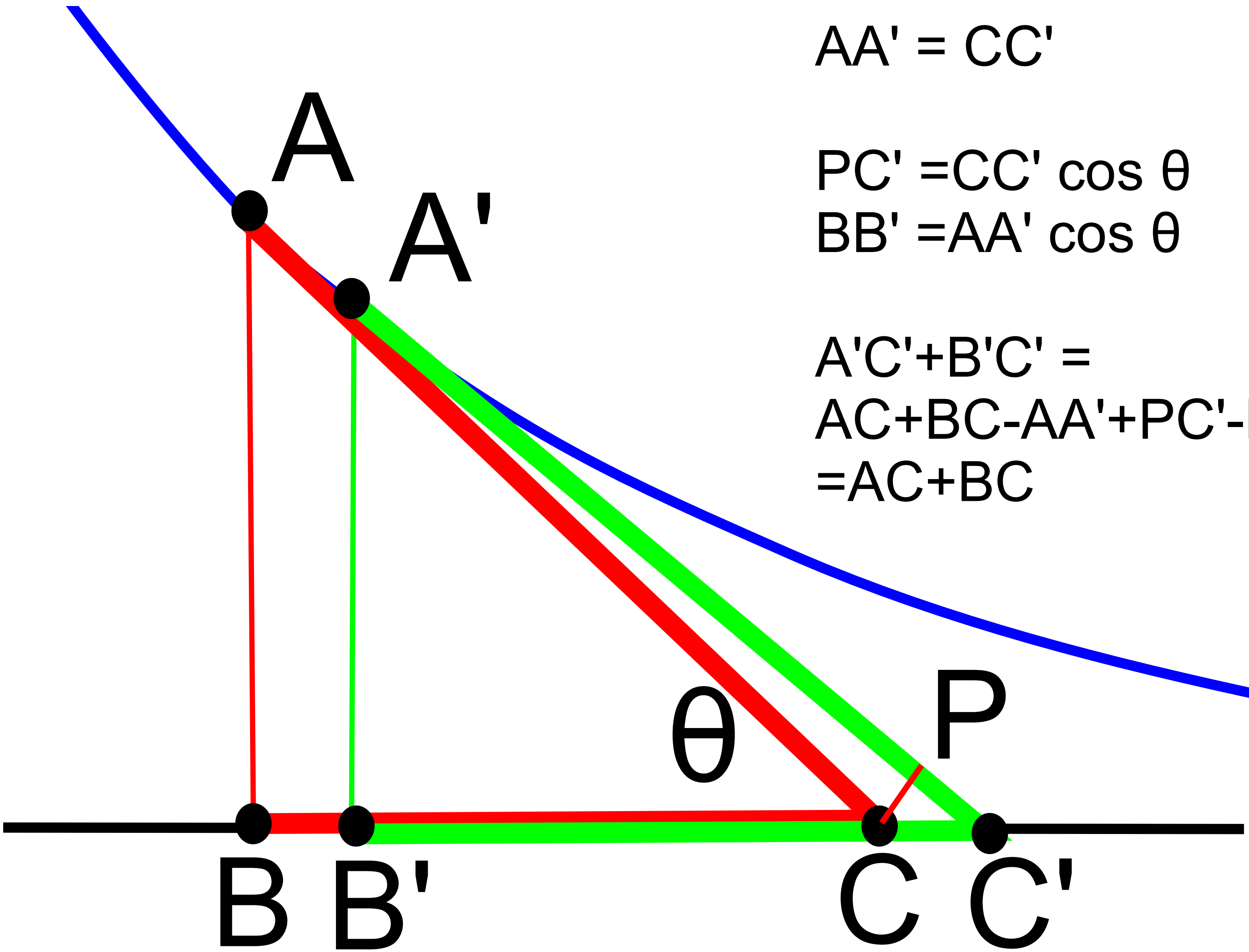
The sweep turns 3 ti
as fast as that of
Cycloid,
3θ/2 instead of θ
Outside area = 3 cir
Cardioid area =
9 – 3 circles = 6 c

Pursuit curve

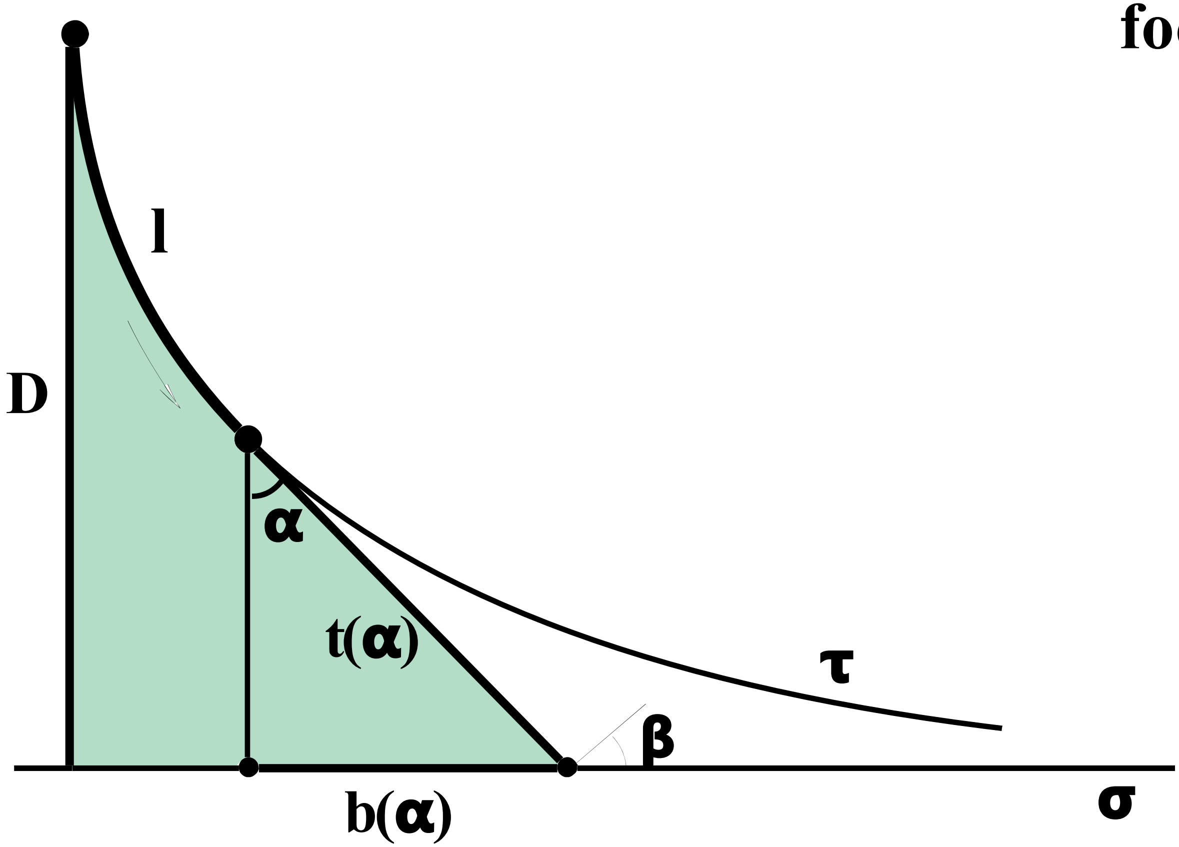
AA' = CC'

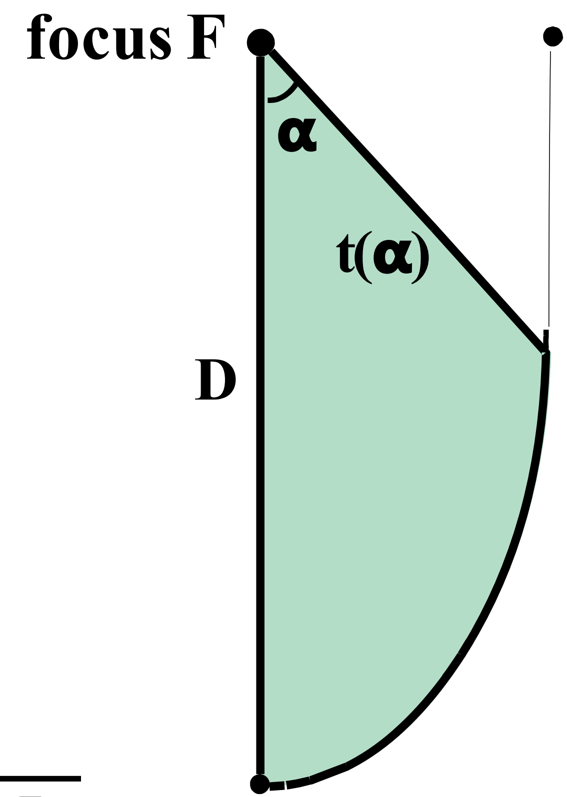
PC' =CC' cos θ
BB' =AA' cos θ

A'C'+B'C' =
AC+BC-AA'+PC'-BB'+CC'
=AC+BC

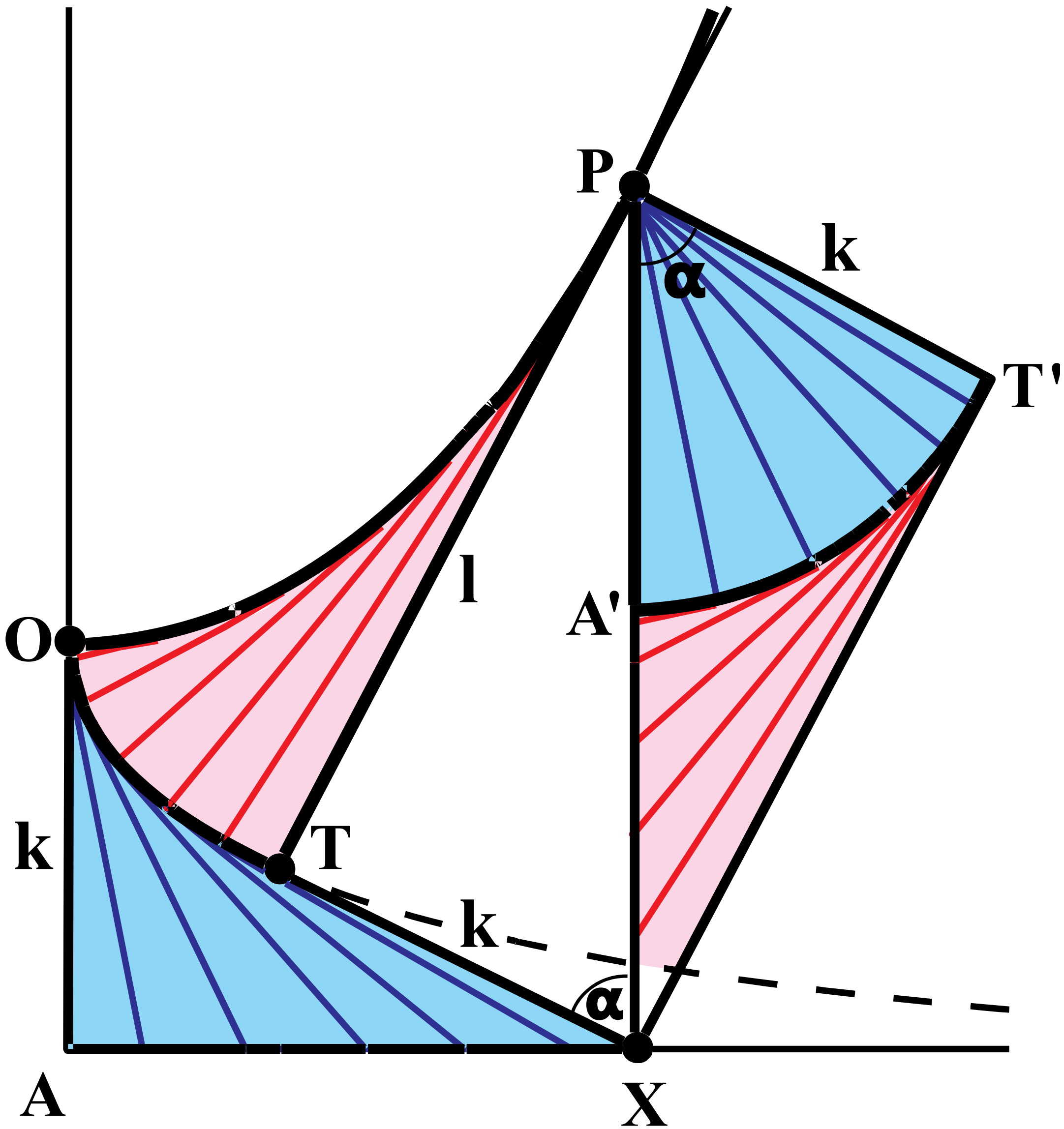


Pursuit curve





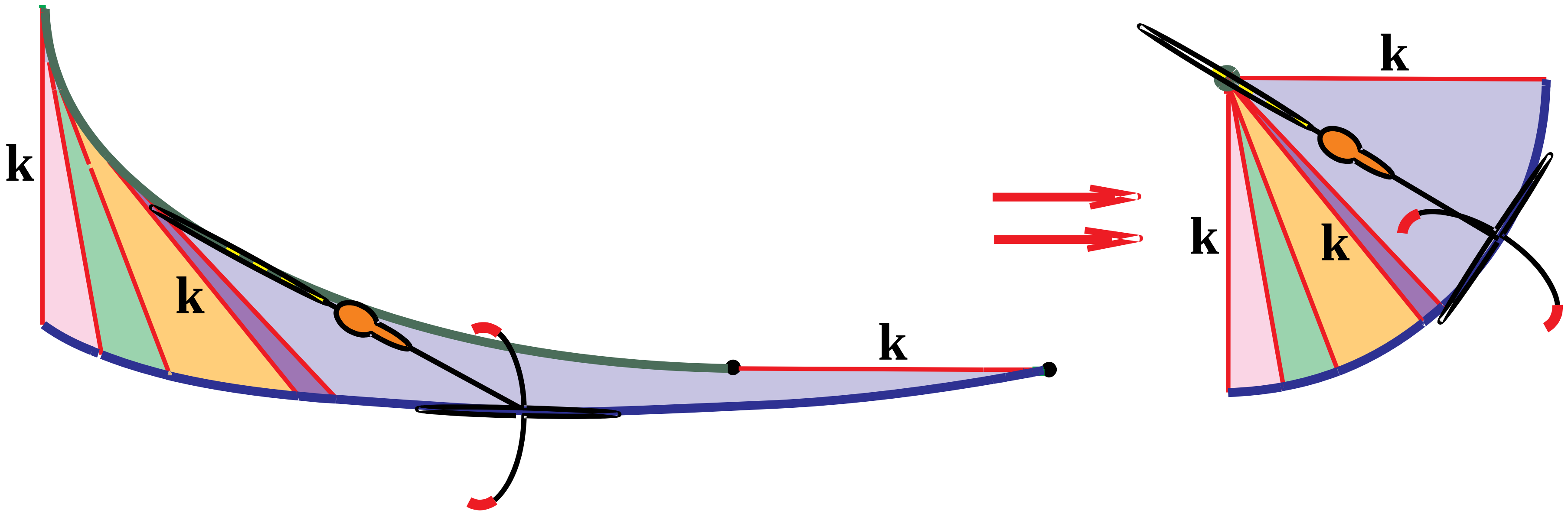
Catenary



The involute of the catenary is the tractrix

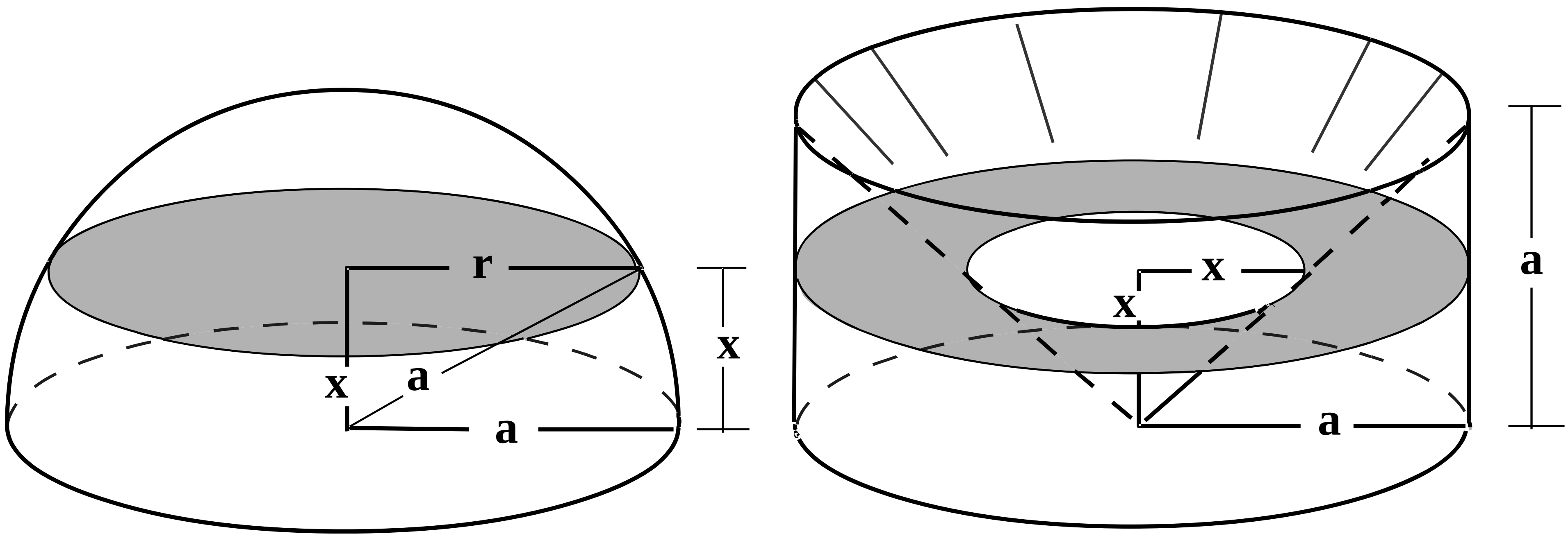
Area = TP.TX = $PX^2.\sin(\alpha).\cos(\alpha)$

And the bicyclix?



Volume of a Sphere

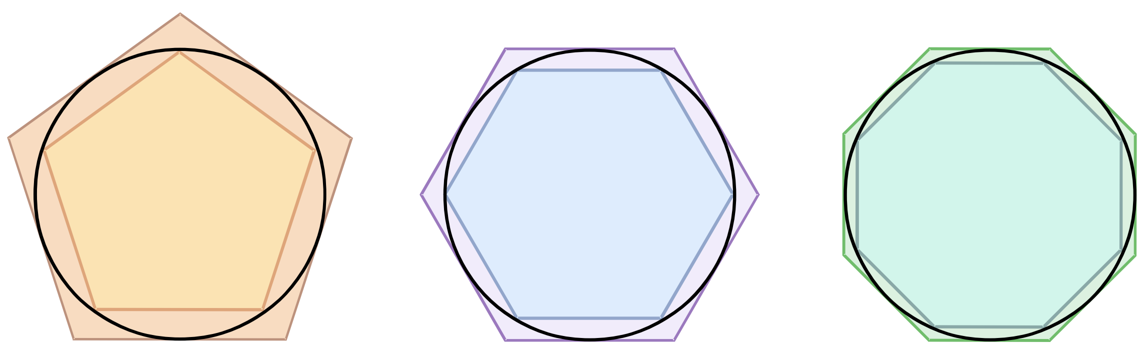
Archimedes had this put on his tombstone



Archimedes c. 287– 212 BC

The first to calculate the value of π rigorously by bounding a circle above and below with regular polygons to give

$$3\frac{10}{71} < \pi < 3\frac{10}{70}$$



He used 96 sided polygons