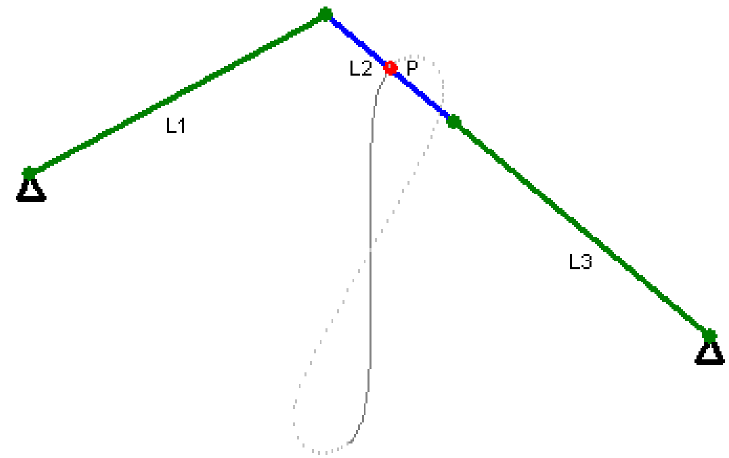
Linkages

David McQuillan

James Watt 1784



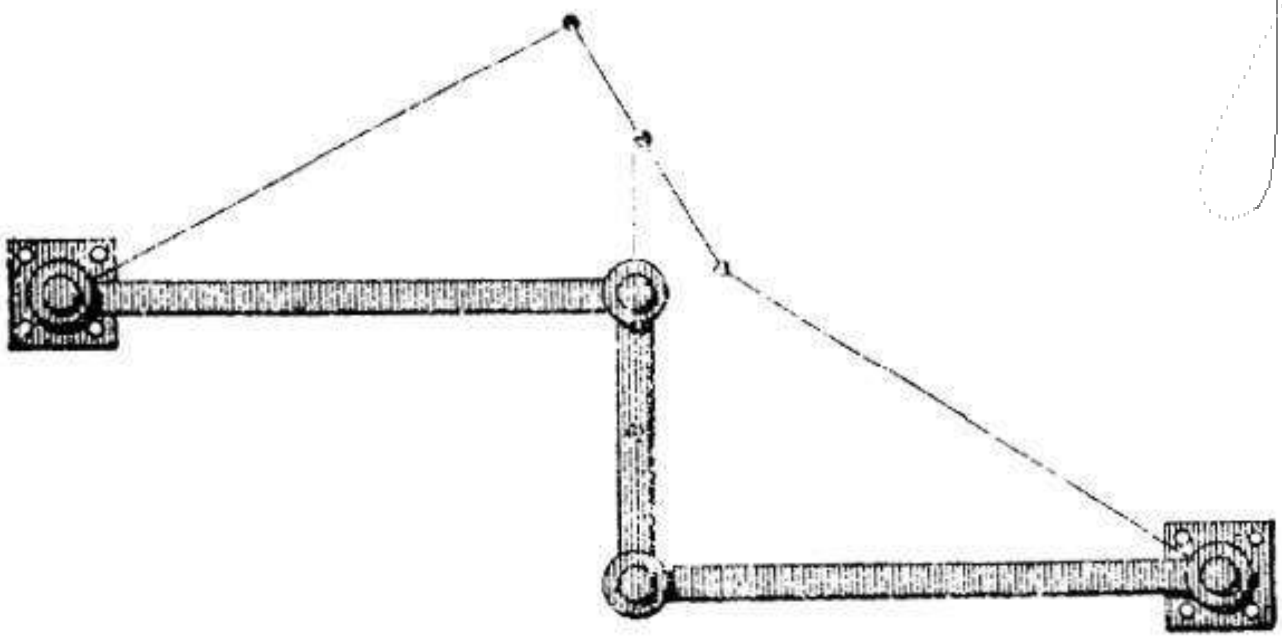
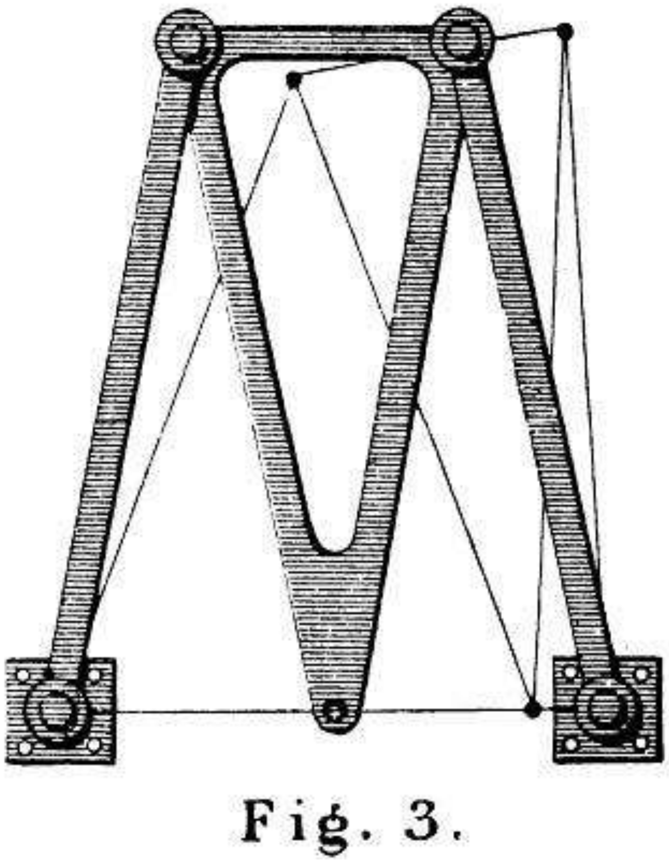
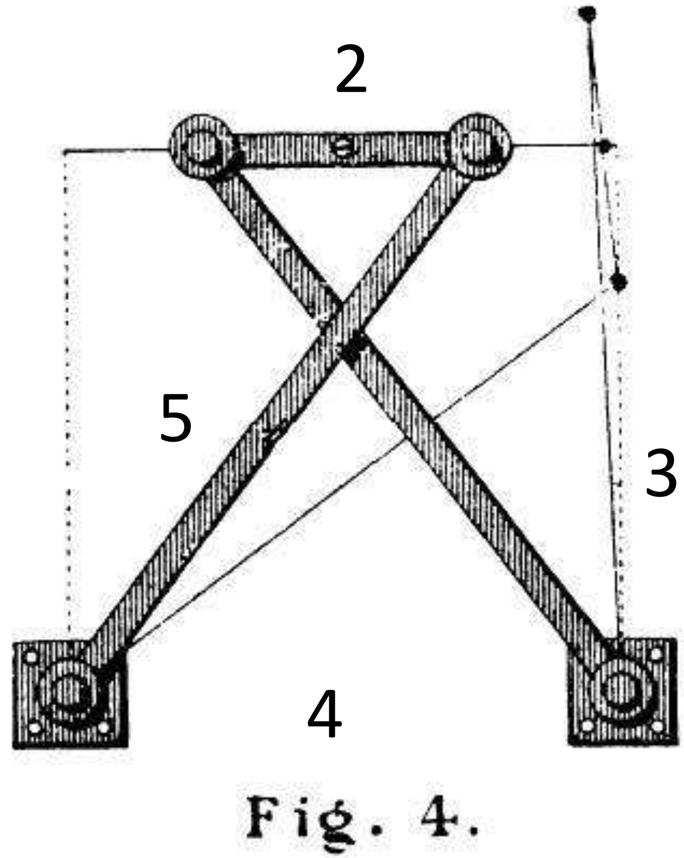


Fig. 2.

More accurate failures



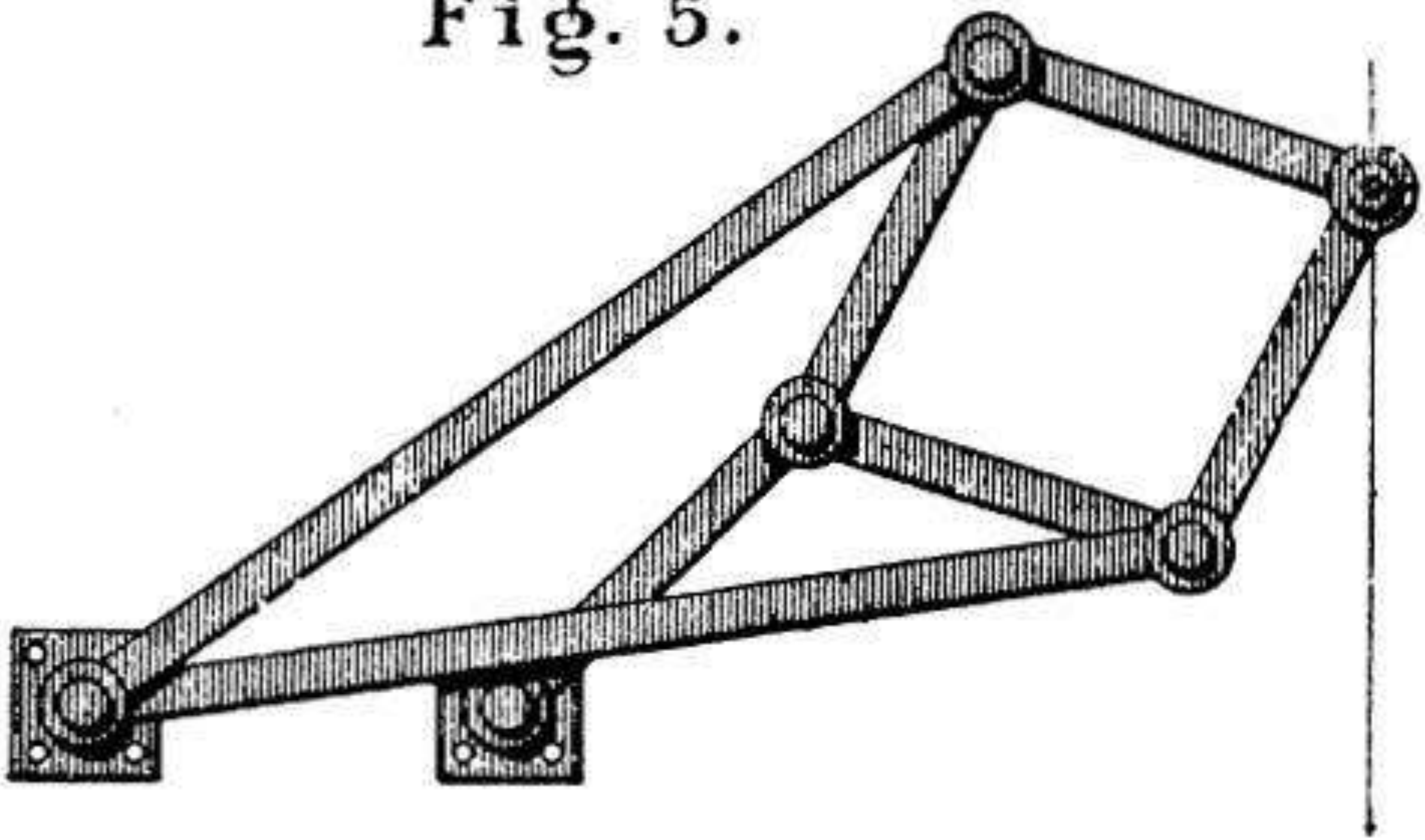
Richard Roberts



Chebyshev

Charles-Nicolas Peaucellier 1864

Fig. 5.



Historical Note

Charles-Nicolas Peaucellier 1832–1913. was a engineer and French army officer.

Rediscovered by

Yom Tov Lipman Lipkin 1846-1876, a Lithuanian Jewish mathematician and inventor.

Popularized in Britain by **James Joseph Sylvester** and later by **Alfred Kempe**

NATURE SERIES.

HOW TO DRAW A STRAIGHT LINE;

A

LECTURE ON LINKAGES.

BY

A. B. KEMPE, B.A.,

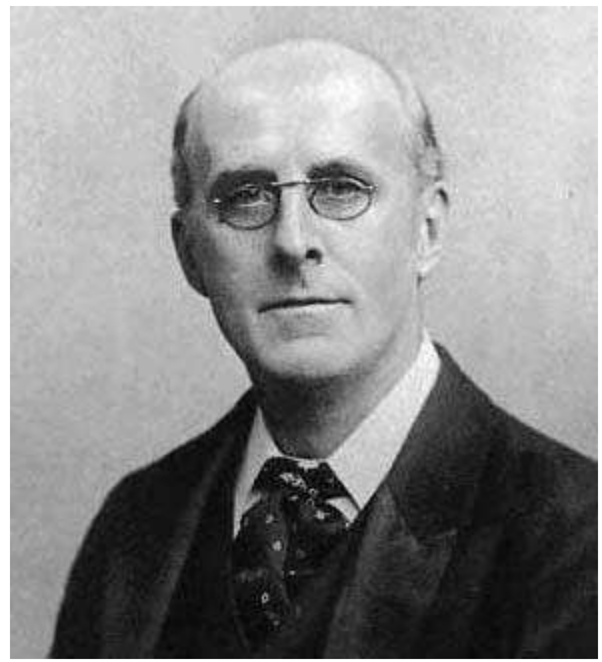
OF THE INNER TEMPLE, ESQ.;
MEMBER OF THE COUNCIL OF THE LONDON MATHEMATICAL SOCIETY;
AND LATE SCHOLAR OF TRINITY COLLEGE, CAMBRIDGE.

WITH NUMEROUS ILLUSTRATIONS.

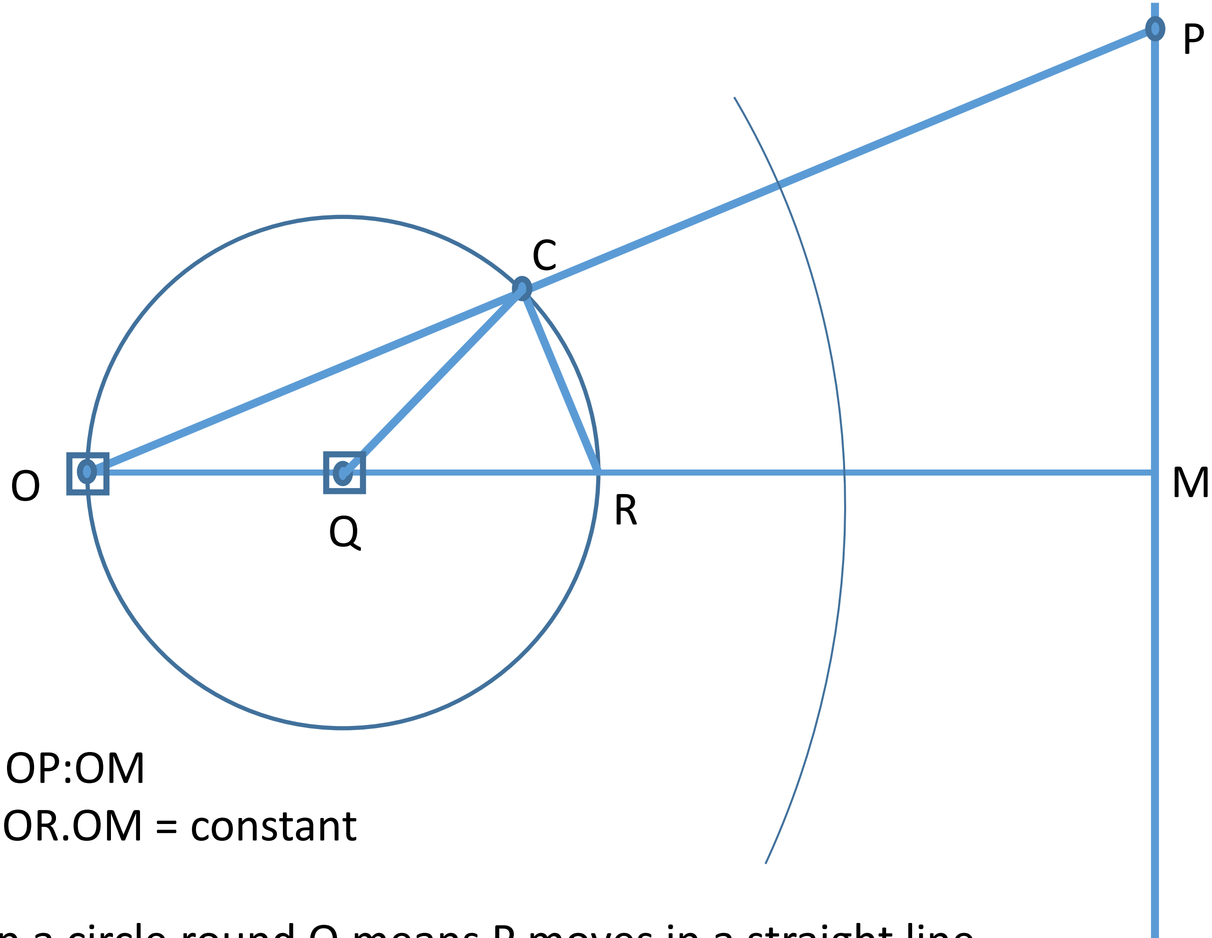
London:
1877.

Lecture on Linkages

Kempe 1877



Inversion of a Circle

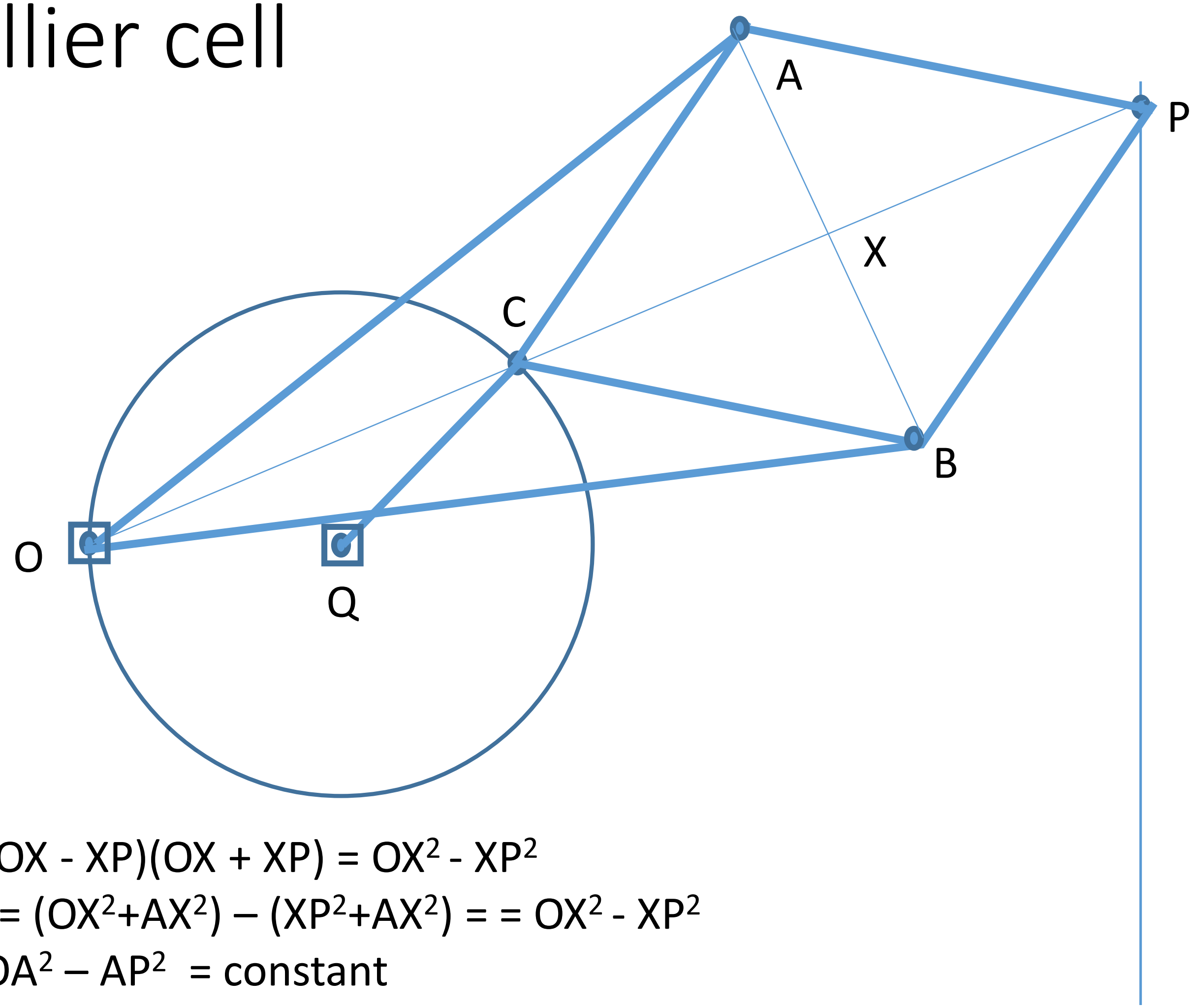


OR:OC = OP:OM

OC.OP = OR.OM = constant

C going in a circle round Q means P moves in a straight line

Peaucellier cell



$OC \cdot OP = (OX - XP)(OX + XP) = OX^2 - XP^2$

$OA^2 - AP^2 = (OX^2+AX^2) - (XP^2+AX^2) = = OX^2 - XP^2$

$OC \cdot OP = OA^2 - AP^2 = \text{constant}$

C going in a circle round Q means P moves in a straight line

Kempe's Universality Theorem

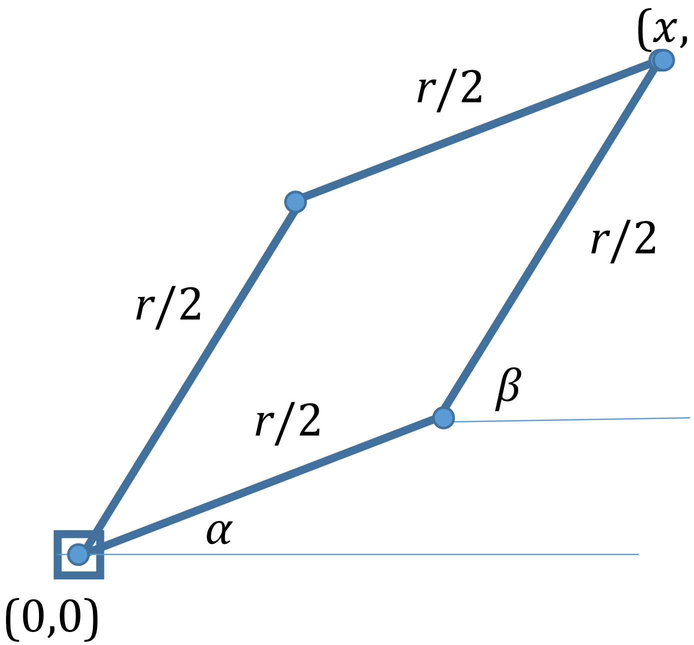
Any algebraic planar curve intersected with a disk gives the allowed locations of a joint of some linkage.

$$\varphi(x, y) = \sum_i c_i x^{p_i} y^{q_i} = 0, \qquad (x-a)^2 + (y-b)^2 \leq R^2$$

Could do it with linkages to do multiplication and addition – but it turns out to be easier to show by turning it into trigonometry!

More recently it has also been done using complex numbers but comes to the same thing.

Stage 1 – Limit it to a disk



$$x = \frac{r}{2}\cos\alpha + \frac{r}{2}\cos\beta$$

$$y = \frac{r}{2}\sin\alpha + \frac{r}{2}\sin\beta$$

$$= \frac{r}{2}\cos\left(\frac{\pi}{2} - \alpha\right) + \frac{r}{2}\cos\left(\frac{\pi}{2} - \beta\right)$$

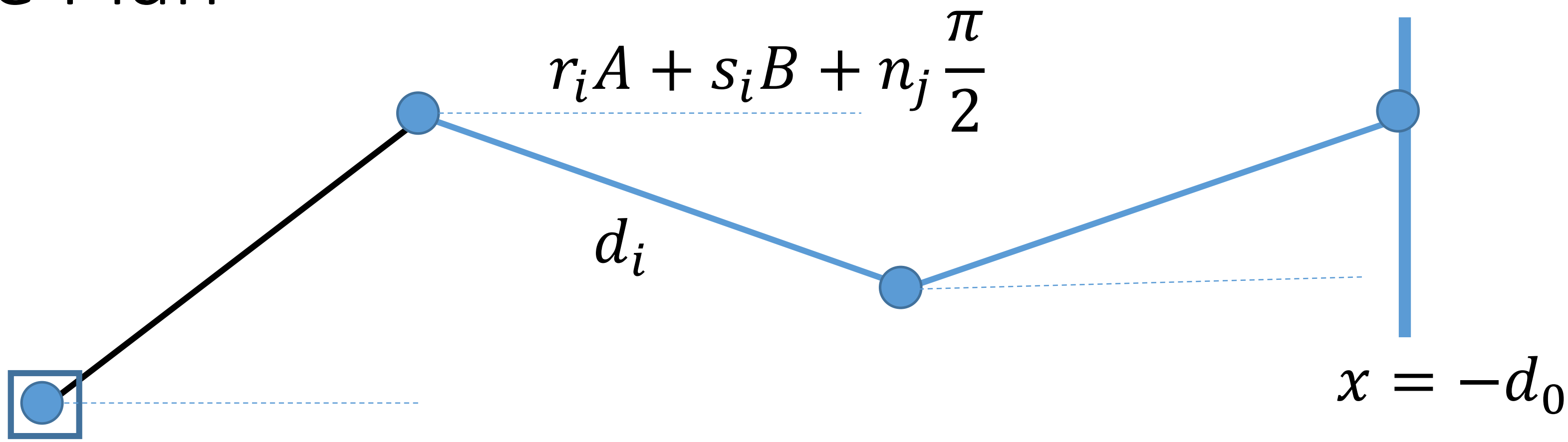
Stage 2 - Multiplication -> addition/subtraction of angles

$$\cos A \times \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$$

$$\varphi(x,y) = \sum_i c_i x^{p_i} y^{q_i} = d_0 + \sum_i d_i \cos\left(r_i A + s_i B + n_j \frac{\pi}{2}\right)$$

This is based on a method called Prosthaphaeresis which was used to do multiplication in the 16th century before logarithms were invented.

The Plan



Construct linkage which is constrained like the equation, then the original (x, y) is the required path.

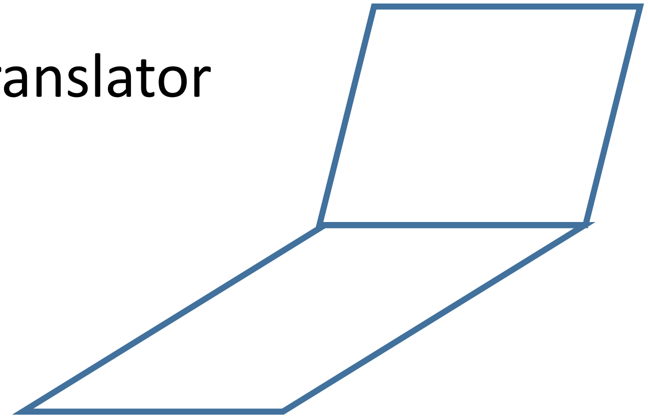
Add or subtract angles

Copy angle from one place to another

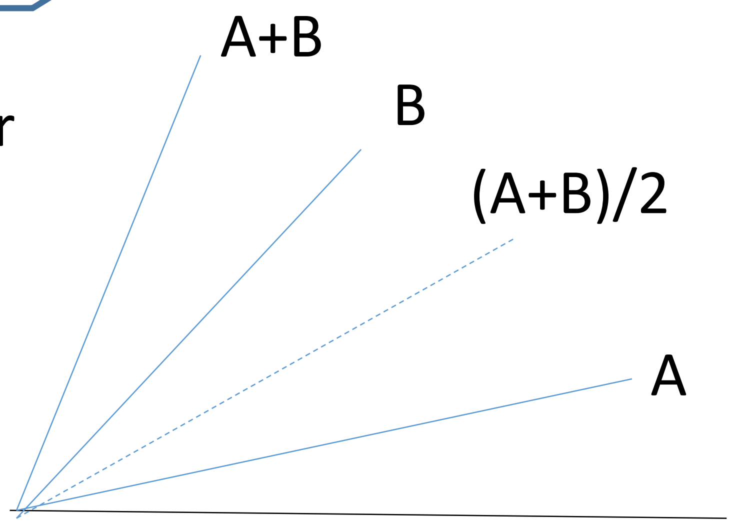
Peaucellier linkage for final constant constraint

The Gadgets

- Translator



*Angle Adder



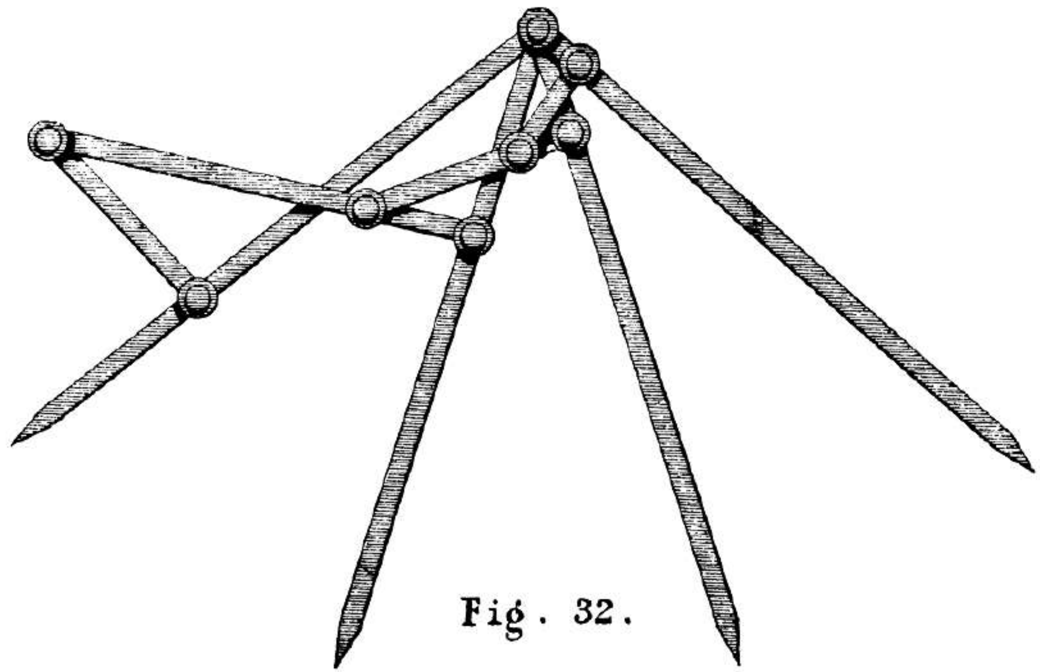
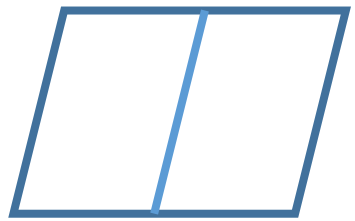


Fig. 32.

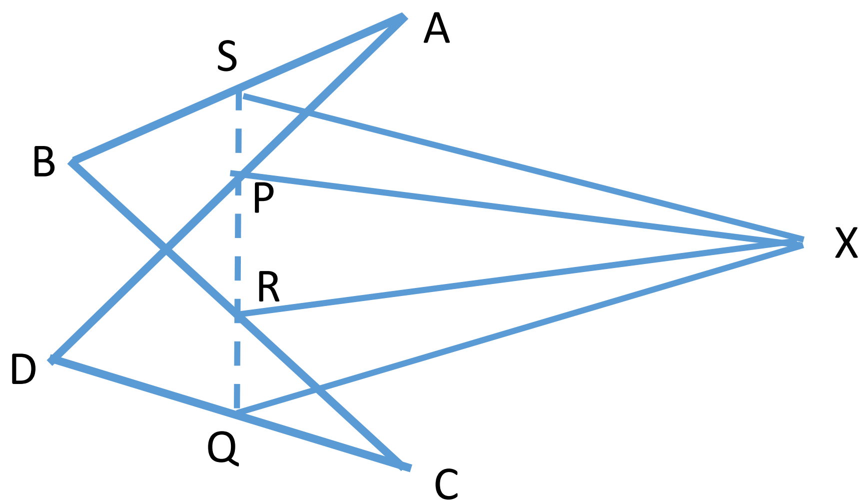
* Angle multiplier

Fixups for Parallelograms

Parallelogram



Contraparallelogram



Don’t allow one to turn into the other

$$XP^2 + AP^2 = XS^2 + AS^2$$

And a ‘use’

Klann Linkage

Odds and Ends

3D - Sarrus Linkage before Peaucellier

Rack and pinion – cannot convert angle linearly to straight line with linkages.